Game Theory: Exercise 6

Games of Incomplete Information and Auctions

Exercise 7 Chapter 10 Harrington.

7. Assume that two countries are on the verge of war and are simultaneously deciding whether or not to attack. A country's military resources are its type, and their relevance is summarized in a parameter which influences the likelihood that they would win a war. Suppose the type space is made up of two values: p' and p'', where $0 < p' < p'' < 1$. A country is type p'' with probability q and type p' with probability $1 - q$. Consider a country of type p (which equals either p' or p''). If it chooses to attack and it attacks first, then it believes it'll win the war with probability xp, where x takes a value such that $p < xp < 1$. If the two countries both attack, then the probability that a type p country wins is p. If a type p country does not attack and the other country does attack, then the probability of victory for the type p country is yp, where y takes a value such that $0 < yp < p$. Finally, if neither country attacks, then there is no war. A country is then more likely to win the war the higher is its type and if it attacks before the other country.

   A country's payoff when there is no war is 0, from winning a war is W, and from losing a war is L. Assume that $W > 0 > L$.

a. Derive the conditions for it to be a symmetric Bayes–Nash equilibrium for a country to attack regardless of its type.

b. Derive the conditions for it to be a symmetric Bayes–Nash equilibrium for a country to attack only if its type is p''.

Exercise 2 – Chapter 26 from Watson.

Two players have to simultaneously and independently decide how much to contribute to a public good. If player 1 contributes x_1 and player 2 contributes x_2 then the value of the public good is $v = 2(x_1 + x_2 + x_1 x_2)$, which they each receive. Assume that x_1 and x_2 are positive numbers. Player 1 must pay a cost x_1^2 of contributing; thus, player 1's payoff in the game is:

$$u_1 = 2(x_1 + x_2 + x_1 x_2) - x_1^2$$

Player 2 pays the cost $t x_2^2$ so that player 2's payoff is:

$$u_2 = 2(x_1 + x_2 + x_1 x_2) - t x_2^2$$

The number t is private information to player 2; player 1 does not observe the precise value of t, but knows that:

t=2 with probability 1/2, and

t=3 with probability 1/2.

Compute the Bayesian Nash equilibrium of this game.

Exercise 2 - Chapter 27 - Watson

Suppose you and one other bidder are competing in a private-value auction. The auction format is sealed bid, first price. Let v and b denote your valuation and bid respectively, let $\hat{v}$ and $\hat{b}$ denote the valuation and bid of your opponent. Your payoff is $(v - b)$ if $b \geq \hat{b}$ and 0 otherwise. Although you do not observe $\hat{v}$, you know that $\hat{v}$ is uniformly distributed over the interval between 0 and 1. That is, v' is the probability that $\hat{v} < v'$. You also know that your opponent bids according to the function $\hat{b}(\hat{v}) = \hat{v}^2$. Suppose your value is 3/5. What is your optimal bid?