Game theory and Supply chain Management

ارائه دهنده: **اشکان حافظ‌الکتب**

اردیبهشت ۹۱

Game theory and Supply chain Management:

- In an operational sense, a supply chain management (SCM) consist of the management of a network of facilities, the exchange of communications, distribution channels, and the firms that procure materials, transform these materials to intermediate and finished products, and distribute the finished products to customer.

- However, in an organizational sense, a supply chain (SC) includes a broad variety of collaborative agreements and contracts among independent enterprises, which integrate them as collaborative networks. These enterprises normally pursue conflicting goals extend across production, purchasing, inventory, transportation and marketing

Game theory and Supply chain Management:

- we emphasize the growing concerns of supply chain management from intra-industry and self-management to include a far greater complexity based on intrinsically more global approaches and the elements that define a supply chain .

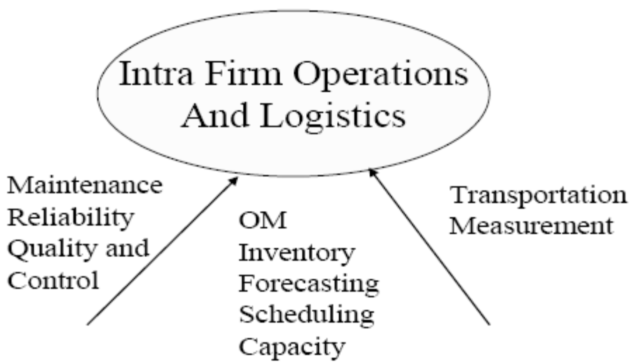


Figure 1.1. Operations Management—Intra Firms Operations and Logistics

Figure 1.2. Operations Management-Intra Firm and Up-stream Collaboration

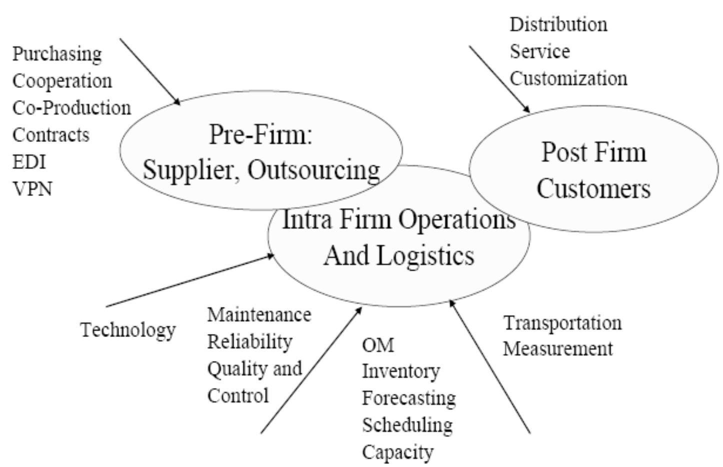


Figure 1.3. Supply Chain: Upstream and Downstream Integration

Game theory and Supply chain Management:

- **Game theory is Multiple-objective Decision making (MODM) with multiple decision makers.**

In Game theory objectives have interconnections with each others.

- **Each decision maker has an objective (or payoff) over specific feasible solution area.**
- **feasible solution for DMs can be separate or common.**

MoDM زیر (بااهداف بی‌مقیاس یا هم‌مقیاس شده) را در نظر می‌گیریم:

$$\max : \{f_1(x), \ldots, f_k(x)\}$$

$$\text{s.t} : x \in S = \{\text{مجموعه محدودیت‌های مشترک}\}$$

Game theory and Supply chain Management:

- **Game theory terminology and its application to Supply chain management**
  - System wide optimal or optimal values for integrated system
  - Nash equilibrium
  - Stackelberg equilibrium
  - Cooperation
  - Potential Coalition
  - Symmetric or Asymmetric information

Game theory and Supply chain Management:

- System wide optimal or optimal values for integrated system

  - When all players integrate and constitute a unit, they have a unique payoff (objective).
  - The optimal value regarding this system is called system wide optimal.
  - In SCM, system wide optimal shows total ideal value that can be achieved, if a complete coalition is performed.
  - Separate optimization of the objectives by players (independent firms) yields a decrease from system wide optimal, which is called double margination effect.

Game theory and Supply chain Management:

- Nash equilibrium (discrete or continues strategies)
  - Discrete case (In zero sum games)

The Gain to Player 1 = The Loss of Player 2

The pay off matrix to B (or a loss to A)

		بازیکن A		
		I	II	III
	۱	−۱	۶	−۲
بازیکن B	۲	۲	۴	۶
	۳	−۲	−۶	۱۲

This problem has a solution, called a (Nash equilibrium) or *saddle-point, because the least* greatest loss to A is equal to the greatest minimum gain to B.
(i.e. maximin(For player B)=minmax(player A)).

When this is the case, the game is said to be stable, and the pay-off table is said to have a saddle-point.

This saddle-point is also called the value of the game, which is the least entry in its row, and the greatest entry in the column.

Game theory and Supply chain Management:

- Nash equilibrium (discrete or continues strategies)
  - Discrete case (In zero sum games)

The Gain to Player 1 = The Loss of Player 2

A بازیکن

I II III

Note that:

1. Not all games can have a pure, single strategy, saddle-point solution for each player.

2. When a game has no saddle point, a solution to the game can be devised by adopting a mixed strategy.

Game theory and Supply chain Management:

- Nash equilibrium (discrete or continues strategies)
  - Discrete case (Non-zero sum games)

The Gain to Player 1 ≠ The Loss of Player 2

B :

A :	B۱	B۲	B۳
A۱	۱۰+,۴	۱+,۵*	۹۸,۴
A۲	۹,۹*	۰,۳	۹۹,۸
A۳	۱,۹۸	۰,۱۰۰*	۱۰۰+,۹۸

There is a unique Nash equilibrium solution (or Saddle point)

B :

A :	B۱	B۲	B۳
A۱	۵+,۰*	−۱,−۱	۱۰+,−۱
A۲	−۱,−۱	۰+,۵*	−۱,−۲
A۳	−۱,۱۰*	−۲,−۱	۶ ,۶*

There is no single Nash equilibrium solution (Saddle point).
Mixed strategies should be used.

Game theory and Supply chain Management:

- Nash equilibrium (discrete or continues strategies)
  - Continues case (An optimization problem (NLP) with multiple variables)

Our challenge in SC optimization with regard to Nash equilibrium include :

1. Existence of Nash equilibrium
2. Unique of Nash equilibrium

Theorem 1.2.3 *Let $f : C \times D \to \mathbb{R}$ be a continuous function. Let $C \subset \mathbb{R}^n$ and $D \subset \mathbb{R}^m$ be convex, closed, and bounded. Suppose that $x \mapsto f(x,y)$ is concave and $y \mapsto f(x,y)$ is convex. Then*

$$v^+ = \min_{y \in D} \max_{x \in C} f(x,y) = \max_{x \in C} \min_{y \in D} f(x,y) = v^-.$$

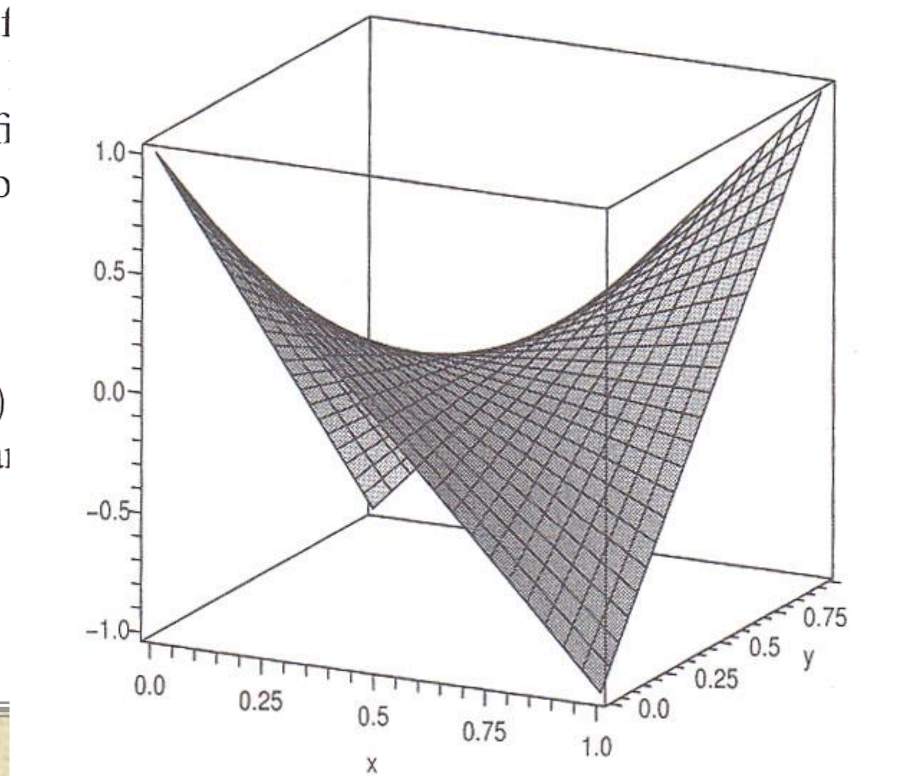
Game theory and Supply chain Management:

- Nash equilibrium (discrete or continues strategies)
  - Continues case

For an example, suppose we look at

$$f(x, y) = 4xy - 2x - 2y + 1 \text{ on } 0 \leq x, y \leq 1.$$

This function y for each x and
concave in x f ıare is closed and
bounded, von ddle point for this
function. To fi ssian for f, which
is the matrix o



Since det(H) ılculus that (x =
$\frac{1}{2}$, $y = \frac{1}{2}$) is aı ːture of f :

Game theory and Supply chain Management:

- Nash equilibrium (discrete or continues strategies)
  - Continues case (An optimization problem (NLP) with multiple variables)

If strategy sets are not constrained and the payoff functions are continuously differentiable. The first-order (necessary) optimality condition results in the following system of two equations in two unknowns y_A^*, y_B^*:

$$\frac{\partial J_A(y_A, y_B^*)}{\partial y_A}\Big|_{y_A = y_A^*} = 0 \text{ and } \frac{\partial J_B(y_A^*, y_B)}{\partial y_B}\Big|_{y_B = y_B^*} = 0 .$$

In addition, the second order (sufficient) optimality condition which ensures that we maximize the payoffs is

$$\frac{\partial^2 J_A(y_A, y_B^*)}{\partial y_A^{\,2}}\Big|_{y_A = y_A^*} < 0 \text{ and } \frac{\partial^2 J_B(y_A^*, y_B)}{\partial y_B^{\,2}}\Big|_{y_B = y_B^*} < 0 .$$

Equivalently, one may determine $y_A^R(y_B) = \arg\max_{y_A \in Y_A}\{J_A(y_A, y_B)\}$ for each $y_B \in Y_B$ to find the best response function, $y_A = y_A^R(y_B)$, of player A and of player B, $y_B = y_B^R(y_A)$ which constitute a system of two equations in two unknowns.

Game theory and Supply chain Management:

- Stackelberg equilibrium

  - Stackelberg strategy is applied when there is an asymmetry in power or in moves of the players.
  - As a result, the decision-making is sequential rather than simultaneous as is the case with Nash strategy.
  - The player who first announces his strategy is considered to be the Stackelberg leader.
  - The follower then chooses his best response to the leader's move.

  - The leader thus has an advantage because he is able to optimize his objective function subject to the follower's best response.

Game theory and Supply chain Management:

- Stackelberg equilibrium

In a two-person game with player A as the leader and player B as the follower, the strategy $y_A^* \in Y_A$ is called a Stackelberg equilibrium for the leader if, for all y_A,

$$J_A(y_A^*, y_B^R(y_A^*)) \geq J_A(y_A, y_B^R(y_A)),$$

where $y_B = y_B^R(y_A)$ is the best response function of the follower.

Definition 2.2 implies that the leader's Stackelberg solution is

$$y_A^* = \arg\max_{y_A \in Y_A} \{J_A(y_A, y_B^R(y_A))\}.$$

That is, if the strategy sets are unconstrained and the payoff functions are continuously differentiable, the necessary optimality condition for the leader is

$$\frac{\partial J_A(y_A, y_B^R(y_A))}{\partial y_A}\bigg|_{y_A = y_A^*} = 0.$$

To make sure that the leader maximizes his profits, we check also the second-order sufficient optimality condition

$$\frac{\partial^2 J_A(y_A, y_B^R(y_A))}{\partial y_A^{\,2}}\bigg|_{y_A = y_A^*} < 0.$$

Game theory and Supply chain Management:

- Cooperation and coordination in supply chain management

  - It is easy to verify that Nash and Stackelberg equilibriums yield objective values lower than system wide optimal.

  - Therefore, each SC utilizes some mechanisms to achieve all or part of system wide optimal when partners work independently.

  - Cooperation has different aspects as follows:

    - Information sharing among SC's partners,
    - Revenue sharing,
    - Discounts,
    - Incentives and rebates,
    - Returns mechanisms,
    - …

Game theory and Supply chain Management:

- Coalition

  - Coalitions form in order to benefit every member of the coalition so that all members might receive more than they could individually on their own.
  - However in coalition mathematics, we try to determine a fair allocation or benefits of cooperation among the player to each member of coalition.
  - A major problem in cooperative game theory is precisely define what fair mean.
  - So that we determine the value of cooperation of each member in a coalition and then the extra value earned by coalition is divided based on these values to the members.

Game theory and Supply chain Management:

- Symmetric or Asymmetric information

- Static and differential games

Game theory and Supply chain Management:

- Cases of Supply chain games

- Pricing games
- Production games
- The stocking games
- The outsourcing games

Game theory and Supply chain Management:

- Pricing games

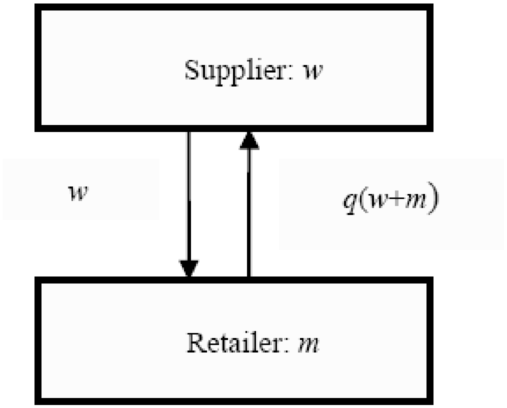


Figure 2.1. Vertical pricing competition

The supplier's problem

$$\max_{w} J_s(w,m) = \max_{w} (w-c)q(w+m)$$

s.t.

$$w \geq c.$$

The retailer's problem

$$\max_{m} J_r(w,m) = \max_{m} mq(w+m)$$

s.t.

$$m \geq 0,$$

$$q(w+m) \geq 0.$$

The centralized problem

$$\max_{m,w} J(m,w) = \max_{m,w} [\, J_r(m,w) + J_s(m,w)] = \max_{m,w} (w+m-c)q(w+m)$$

s.t.

$$m \geq 0,\ q(w+m)\ \geq 0.$$

Game theory and Supply chain Management:

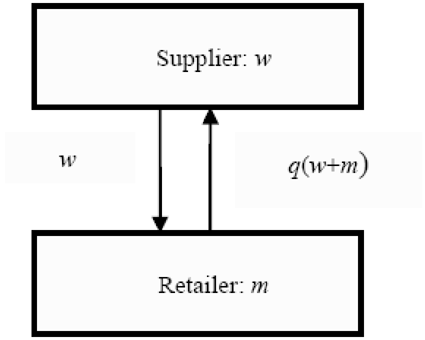


Figure 2.1. Vertical pricing competition

- Pricing games

System-wide optimal solution

We first study the centralized problem by employing the first-order optimality conditions

$$\frac{\partial J(m,w)}{\partial m} = q(w+m) + (w+m-c)\frac{\partial q(p)}{\partial p} = 0,$$

$$\frac{\partial J(m,w)}{\partial w} = q(w+m) + (w+m-c)\frac{\partial q(p)}{\partial p} = 0.$$

$$q(p^*) + (p^* - c)\frac{\partial q(p^*)}{\partial p} = 0.$$

$$p^* = w + m$$

$$\frac{\partial^2 J(p)}{\partial p^2} = \frac{\partial q(p)}{\partial p} + \frac{\partial q(p)}{\partial p} + (p-c)\frac{\partial^2 q(p)}{\partial p^2} < 0,$$

Game theory and Supply chain Management:

- Pricing games

Game analysis

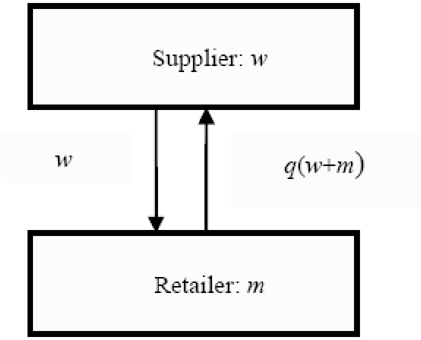


Figure 2.1. Vertical pricing competition

$$\frac{\partial J_r(m,w)}{\partial m} = q(w+m) + m\frac{\partial q(p)}{\partial p} = 0\ .$$

$$\frac{\partial J_s(m,w)}{\partial w} = q(w+m) + (w-c)\frac{\partial q(w+m)}{\partial p} = 0\ .$$

It is straightforward to show that both supplier's and retailer's payoff strictly concave on *w* and *m*, respectively.

As a result the Nash equilibrium point are achieved by Solving two above equilibrium which result in:

$$w\text{-}c\text{-}m=0 \text{ and } q(c+2m) + m\frac{\partial q(c+2m)}{\partial p} = 0\ .$$

Game theory and Supply chain Management:

- Pricing games

Stackelberg game analysis

Next, we assume that the supplier makes the first move by setting the wholesale price. The retailer then decides on what price to set and, hence, the quantity to order.

$J_s(m,w)=(w\text{-}c)q(w+m^R(w))$.

Differentiating the supplier's objective function we have

$$\frac{\partial J_s(m,w)}{\partial w} = q(w+m^R(w)) + (w-c)\frac{\partial q(w+m)}{\partial p}\frac{\partial m^R(w)}{\partial w} = 0\,,$$

where $\dfrac{\partial m^R(w)}{\partial w}$ is determined by differentiating (2.8) with m set equal to $m^R(w)$.

$$\frac{\partial q(w+m)}{\partial p}\left(1+\frac{\partial m^R(w)}{\partial w}\right) + \frac{\partial m^R(w)}{\partial w}\frac{\partial q(p)}{\partial p} + m\frac{\partial^2 q(p)}{\partial p^2}\left(1+\frac{\partial m^R(w)}{\partial w}\right) = 0\,.$$

Thus

$$\frac{\partial m^R(w)}{\partial w} = -\left(\frac{\partial q(w+m)}{\partial p} + m\frac{\partial^2 q(w+m)}{\partial p^2}\right)\Bigg/\left(\frac{\partial q(w+m)}{\partial p} + \frac{\partial q(w+m)}{\partial p} + m\frac{\partial^2 q(w+m)}{\partial p^2}\right). \quad (2.16)$$

Game theory and Supply chain Management:

Pricing games

Stackelberg game analysis

we conclude that a pair (w_s, m_s) constitutes a Stackelberg equilibrium of the pricing game if there exists a joint solution in *w* and *m* of the following equations

$$q(w+m)+(w-c)\frac{\partial q(w+m)}{\partial p}\frac{\partial m}{\partial w}=0\ ,$$

$$q(w+m)+m\frac{\partial q(w+m)}{\partial p}=0\ ,$$

where

$$\frac{\partial m}{\partial w}=-\left(\frac{\partial q(w+m)}{\partial p}+m\frac{\partial^2 q(w+m)}{\partial p^2}\right)\Bigg/\left(\frac{\partial q(w+m)}{\partial p}+\frac{\partial q(w+m)}{\partial p}+m\frac{\partial^2 q(w+m)}{\partial p^2}\right)$$

Game theory and Supply chain Management:

- Pricing games

An example

Let the demand be linear in price, $q(p)=a-bp$ and the supplier's cost negligible, $c=0$. Thus we obtain the problem solved in Example 2.1. Note that the demand requirements, $\frac{\partial q}{\partial p}=-b<0$ and $\frac{\partial^2 q}{\partial p^2}\leq 0$ are met for the selected function. Using Proposition 2.2. we solve (2.15),

$$q(2m^n)+m^n\frac{\partial q(2m^n)}{\partial p}=a-b2m^n+m^n(-b)=0,\ w^n=m^n$$

to find Nash equilibrium $w^n=m^n=\frac{a}{3b}$, hence, $p^n=w^n+m^n=\frac{2a}{3b}$ and $q(p^n)=\frac{a}{3}$, as is also the case in Example 2.1. The payoff for the equilibrium is identical for both players, $J_r(m^n,w^n)=J_s(m^n,w^n)=\frac{a^2}{9b}$.

Game theory and Supply chain Management:

- Pricing games

An example

verify that the Stackelberg solution is the same as in Example 2.2,

$$w^s = \frac{a}{2b},\; m^s = \frac{a}{4b},\; p^s = w^s + m^s = \frac{3a}{4b},\; q(p^s) = \frac{a}{4},$$

$$J_s(m^s, w^s) = \frac{a^2}{8b} \text{ and } J_r(m^s, w^s) = \frac{a^2}{16b}.$$

Finally, the centralized solution (2.7) (see also Example 2.3) is

$$q(p^*) + (p^* - c)\frac{\partial q(p^*)}{\partial p} = a - bp^* + p^*(-b) = 0,$$

that is,

$$m^* + w^* = p^* = \frac{a}{2b},\; q(p^*) = \frac{a}{2} \text{ and } J(p^*) = \frac{a^2}{4b}.$$

Game theory and Supply chain Management:

- Pricing games

An example

Conclusions:

1. Final price of products increase due to vertical competition between supplier and retailer.
2. Total demand of market decrease due to vertical competition between supplier and retailer.
3. These effect is called double margination.
4. Double margination deteriorates supply chain efficiency.
5. Cooperation between supplier and retailer can improve SC's efficiency.

Game theory and Supply chain Management:

- Pricing games

- we observe that the supplier ignores the retailer's margin m when setting the wholesale price. The remaining question is how to induce the retailer to order more, or the supplier to reduce the wholesale price, i.e., how to coordinate the supply chain and thus increase its total profit.
- Of course, the supplier may set the wholesale price at his marginal cost, w=c, or the retailer may set his margin at zero. In this situation the SC is perfectly coordinated.
- However, the supply chain member who gives up his margin gets no profit at all. The most popular way of dealing with such a problem is by discounting or by collaboration for profit sharing.
- Profit sharing can obtained by Fixed fee charge by retailer.

Game theory and Supply chain Management:

- Production games
- Now we shall study the effect of horizontal production competition

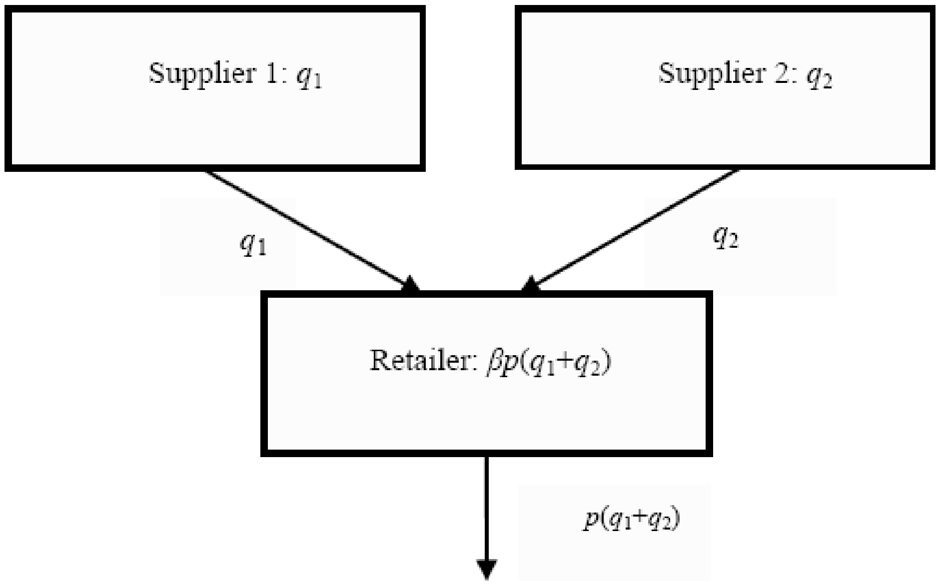


Figure 2.3. Horizontal competition for the same retailer

- Consider two manufacturers producing the same or substitutable types of product over a period of time and thus competing horizontally for the same customers, possibly for the same retailer.
- Accordingly, the manufacturers are suppliers with ample capacity and the order period is longer than the supplier's lead-time.

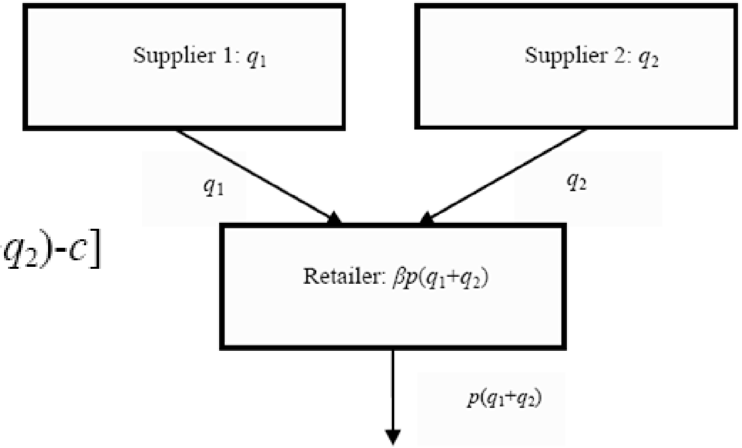
Game theory and Supply chain Management:

- Production games

- we assume that the retail price is a function of customer demand which is referred to as Cournot's model of production competition. Specifically, the product is characterized by an endogenous

- price function of total demand $Q=q_1+q_2$, *$p=p(Q)$, which, since the products* are fully substitutable, is symmetric in q_1 *and* q_2. *We assume that this* symmetric function is down-sloping (concave) in the total quantity of the products, i.e. $\partial p/\partial q_1, \partial p/\partial q_2<0$, and concave i.e. $\frac{\partial^2 p}{\partial {q_1}^2}, \frac{\partial^2 p}{\partial {q_2}^2}, \frac{\partial^2 p}{\partial q_1 \partial q_2} < 0$

Game theory and Supply chain Management:



The problem of supplier 1

$$\max_{q_1} J_1(q_1,q_2) = \max_{q_1} \; q_1[p(q_1+q_2)-c]$$

s.t.

$$q_1 \geq 0,\ p(q_1+q_2) \geq c.$$

The problem of Supplier 2

$$\max_{q_2} J_2(q_1,q_2) = \max_{q_2} \; q_2[p(q_1+q_2)-c]$$

s.t.

$$q_2 \geq 0,\ p(q_1+q_2) \geq c,$$

The centralized problem

$$\max_{q_1,q_2} J(q_1,q_2) = \max_{q_1,q_2} [J_1(q_1,q_2)+J_2(q_1,q_2)] =$$

$$\max_{q_1,q_2} \; q_1[p(q_1+q_2)-c] + q_2[p(q_1+q_2)-c]$$

s.t.

$$q_1 \geq 0,\ q_2 \geq 0,\ p(q_1+q_2) \geq c.$$

Game theory and Supply chain Management:

- Production games

System-wide optimal solution

Define Q' so that $p(Q')=c$. Then it is easy to verify that,

$$\frac{\partial^2 J}{\partial q_1{}^2} = \frac{\partial^2 J}{\partial q_2{}^2} = \frac{\partial^2 J}{\partial q_1 \partial q_2} = 2\frac{\partial p}{\partial Q} + q_1 \frac{\partial^2 p}{\partial Q^2} + q_2 \frac{\partial p^2}{\partial Q^2} < 0 .$$

This implies that the Hessian of $J(q_1,q_2)$ is semi-definite negative and thus the function $J(q_1,q_2)$ is jointly concave in production quantities q_1 and q_2 for $q_1 + q_2 \in [0, Q']$.

total order Q matters in terms of optimality. Considering the symmetric solution to the above system of equations as well, $q^* = q_1^* = q_2^*$, we obtain the following equation

$$p(2q^*) - c + 2q^* \frac{\partial p(2q^*)}{\partial Q} = 0 . \qquad (2.20)$$

Game theory and Supply chain Management:

- Production games

Game analysis

Consider now a decentralized supply chain characterized by non-cooperative firms and assume that both players simultaneously decide how many products to produce and supply to the retailer. Using the first-order optimality conditions for the suppliers' problems we find

$$\frac{\partial J(q_1,q_2)}{\partial q_1} = p(q_1+q_2) - c + q_1\frac{\partial p(q_1+q_2)}{\partial q_1} = 0,$$

$$\frac{\partial J(q_1,q_2)}{\partial q_2} = p(q_1+q_2) - c + q_2\frac{\partial p(q_1+q_2)}{\partial q_2} = 0\,.$$

Again, since the two problems are symmetric, the competition is symmetric. That is, the solution to this system of equations is $q = q_1 = q_2$, which satisfies the following equation

$$p(2q) - c + q\frac{\partial p(2q)}{\partial Q} = 0. \qquad (2.21)$$

Game theory and Supply chain Management:

- Production games

Proposition 2.4. *In horizontal competition of the production game with equal power players, the retail price will be lower and the quantities produced by the manufacturers higher than the system-wide optimal price and production quantity respectively.*

Proof: Comparing (2.21) and (2.20) we observe that if $q=q^*$, then

$$p(2q) - c + q\frac{\partial p(2q)}{\partial Q} > p(2q^*) - c + 2q^*\frac{\partial p(2q^*)}{\partial Q} = 0,$$

while the derivative of the left-hand side of this inequality with respect to q is negative. Thus, $q>q^*$, which, in regard to the down-sloping price function $p(2q)$, means that $p(2q)<p(2q^*)$.

Game theory and Supply chain Management:

Stackelberg solution

Next we assume that one of the suppliers is the leader, say supplier-one. To find the Stackelberg equilibrium, we need to maximize supplier-one's objective with q_1, subject to the best supplier-two's response $q_2 = q_2^R(q_1)$. Let $q_2 = q_2^R(q_1)$ satisfy the following equation

$$p(q_1 + q_2) - c + q_2 \frac{\partial p(q_1 + q_2)}{\partial q_2} = 0. \qquad (2.23)$$

The Stackelberg equilibrium is determined by maximizing the following function

$$\max_{q_1} J_1(q_1) = \max_{q_1} \; q_1[p(q_1 + q_2^R(q_1)) - c].$$

Differentiating this function we find

$$\frac{\partial J_1(q_1)}{\partial q_1} = p(q_1 + q_2^R(q_1)) - c + q_1 \frac{\partial p(q_1 + q_2^R(q_1))}{\partial Q}\left(1 + \frac{\partial q_2^R(q_1)}{\partial q_1}\right) = 0\,, \quad (2.24)$$

where $\frac{\partial q_2^R(q_1)}{\partial q_1}$ is determined by differentiating (2.23) with q_2 set equal to $q_2^R(q_1)$

$$\frac{\partial p(Q)}{\partial Q}\left(1 + \frac{\partial q_2^R(q_1)}{\partial q_1}\right) + \frac{\partial q_2^R(q_1)}{\partial q_1}\frac{\partial p(Q)}{\partial Q} + q_2^R(q_1)\frac{\partial p^2(Q)}{\partial Q^2}\left(1 + \frac{\partial q_2^R(q_1)}{\partial q_1}\right) = 0\,.$$

Thus

$$\frac{\partial q_2^R(q_1)}{\partial q_1} = -\left(\frac{\partial p(Q)}{\partial Q} + q_2^R(q_1)\frac{\partial p^2(Q)}{\partial Q^2}\right) \Bigg/ \left(2\frac{\partial p(Q)}{\partial Q} + q_2^R(q_1)\frac{\partial p^2(Q)}{\partial Q^2}\right). \quad (2.25)$$

Game theory and Supply chain Management:

- Production games
- A numerical example

Let the price be linear in production quantity, $p=a-bQ$, $Q=q_1+q_2$, $p(0)=a>c$. Note that the price requirements, $\frac{\partial p}{\partial q_1}=\frac{\partial p}{\partial q_2}=-b<0$ and $\frac{\partial^2 p}{\partial q_1{}^2}=\frac{\partial^2 p}{\partial q_2{}^2}=\frac{\partial^2 p}{\partial q_1 \partial q_2}=0$ are met for the selected function. Using Proposition 2.5 we solve (2.22),

$$p(2q^n)-c+q^n\frac{\partial p(2q^n)}{\partial Q}=a-2bq^n-c+q^n(-b)=0$$

and find that $q_1{}^n = q_2{}^n = \frac{a-c}{3b}$, hence, $p^n=\frac{1}{3}a+\frac{2}{3}c$. The payoffs for the equilibrium are thus identical for both players, $J_1(q_1{}^n,q_2{}^n)=J_2(q_1{}^n,q_2{}^n)=\frac{(a-c)^2}{9b}$.

Game theory and Supply chain Management:

- Production games
- A numerical example

Based on (2.23) we can identify the best response function of the second supplier

$$p(q_1+q_2)-c+q_2\frac{\partial p(q_1+q_2)}{\partial q_2}=a-b(q_1+q_2)-c+q_2(-b)=0,$$

and thus

$$q_2=q_2^r(q_1)=\frac{a-bq_1-c}{2b}.$$

This response is then employed in (2.24) and (2.25) to find the Stackelberg equilibrium. Equivalently, by substituting this response into the first supplier objective function

$$\max_{q_1}\ q_1[p(q_1+q_2^R(q_1))-c]=\max_{q_1}\ q_1[\frac{a}{2}-\frac{bq_1}{2}-\frac{c}{2}].$$

and using the first-order optimality conditions, we obtain an explicit resolution of equation (2.24) for our example,

$$\frac{\partial J_1}{\partial q_1}=[\frac{a}{2}-\frac{bq_1}{2}-\frac{c}{2}]+q_1[-\frac{b}{2}]=0.$$

Accordingly, $q_1^s=\frac{a-c}{2b}$, $q_2^s=\frac{a-c}{4b}$, $p^s=\frac{a+3c}{4}$, $J_1(q_1^s,q_2^s)=\frac{(a-c)^2}{8b}$ and $J_2(q_1^s,q_2^s)=\frac{(a-c)^2}{16b}$. Note that instead of equal payoff under a simultaneous Nash strategy, the first supplier, who is the leader, gains a profit which is twice as much as the follower's profit under a sequential Stackelberg strategy.

Game theory and Supply chain Management:

- Production games
- A numerical example

Finally, the centralized solution (2.20) is

$$p(2q^*) - c + 2q^* \frac{\partial p(2q^*)}{\partial Q} = a - 2bq^* - c + 2q^*(-b) = 0\,.$$

Or, $q_1^* = q_2^* = \dfrac{a-c}{4b}$, hence, $p^* = \dfrac{1}{2}a + \dfrac{1}{2}c$ and the system-wide optimal supply chain profit is $J(q_1^*, q_2^*) = \dfrac{(a-c)^2}{4b}$.

$$q_1^s = \frac{a-c}{2b} > q_1^n = \frac{a-c}{3b} > q_1^* = \frac{a-c}{4b}\,.$$

$$J_1(q_1^s, q_2^s) + J_2(q_1^s, q_2^s) = \frac{3(a-c)^2}{16b} < J_1(q_1^n, q_2^n) + J_2(q_1^n, q_2^n) =$$

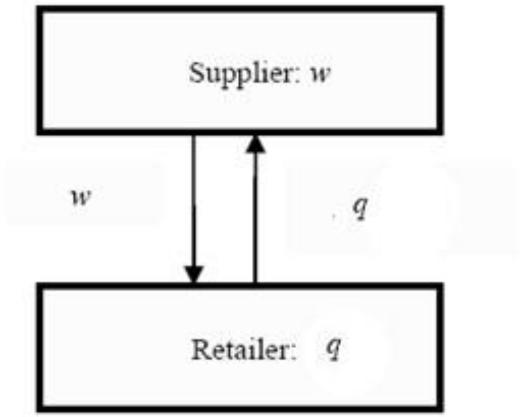
$$\frac{2(a-c)^2}{9b} < J(q_1^*, q_2^*) = \frac{(a-c)^2}{4b}\,.$$

Game theory and Supply chain Management:

- STOCKING COMPETITION WITH RANDOM DEMAND

- In contrast to the previous section, we now assume that the retailer demand is random and proceed to adapt two classic newsvendor models into two stocking/pricing games.
- In one game the supplier sets the wholesale price to sell some of his stock while the retailer decides on the quantity to purchase in order to replenish his stock.

- We refer to this game as the stocking game.



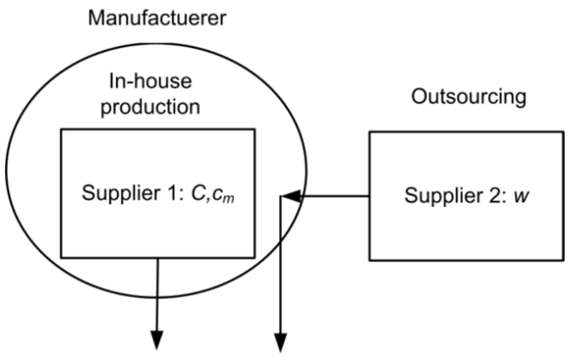
the stocking game

- `

Game theory and Supply chain Management:

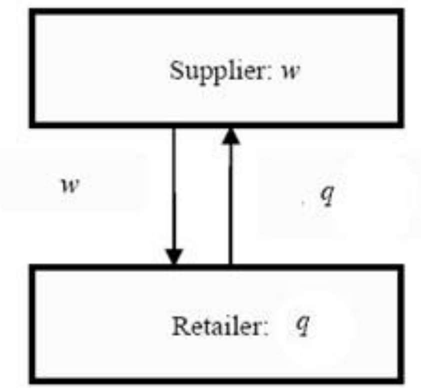
- STOCKING COMPETITION WITH RANDOM DEMAND

- The other game is related to a manufacturer who pays a setup cost for each production order.
- To avoid this irreversible cost, the manufacturer has the alternative of outsourcing current in-house production to a supplier.
- Similar to the stocking game, the supplier decides on the wholesale price and does not charge a fixed order cost. Unlike the stocking game, the manufacturer determines first whether to outsource the production at this wholesale price or to produce in-house and then determining the proper quantity to order.
- We refer to this game as the outsourcing game.



Game theory and Supply chain Management:

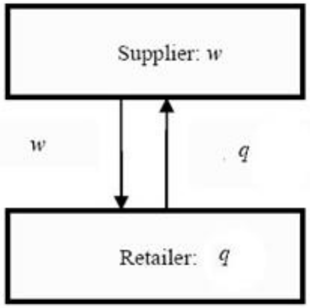
- STOCKING COMPETITION WITH RANDOM DEMAND
- The stocking game.
- The classical, single-period, newsboy or newsvendor problem formulation assumes random exogenous demand, *d*.
- If the retailer orders less than the demand at the end of period, then shortage h^- cost per unit of unsatisfied demand is incurred.
- if the retailer orders more than he is able to sell, unit inventory cost h^+ (mitigated by salvage cost) is incurred for units left over at the end of period.



- The retailer goal is to find order quantity, *q*, to maximize expected overall profits.
- The supplier goal is to choose a wholesale price, *w*, to maximize expected overall profits.

Game theory and Supply chain Management:

- The stocking game.



The retailer's problem

$$\max_q J_r(q,w) = \max_q \{E[ym - h^+x^+ - h^-x^-] - wq\},$$

s.t.

$$x = q - d,$$

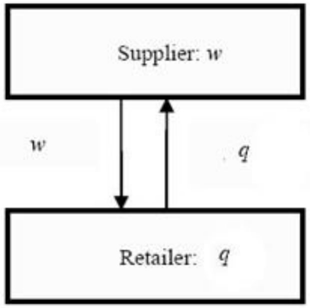
$$q \geq 0,$$

where $x^+ = \max\{0, x\}$ and $x^- = \max\{0, -x\}$ are inventory surplus and shortage at the end of selling season respectively, and $y = \min\{q, d\}$ is the number of products sold.

$$\max_q J_r(q,w) = \max_q \{$$

$$\int_0^q mDf(D)dD + \int_q^\infty mqf(D)dD - \int_0^q h^+(q-D)f(D)dD - \int_q^\infty h^-(D-q)f(D)dD - wq\},$$

Game theory and Supply chain Management:



- The stocking game.

The supplier's problem

$$\max_{w} J_s(q,w) = (w-c)q$$

s.t.

$$c \leq w \leq w^M.$$

- The corresponding centralized problem is based on the sum of two objective functions of retailer and suppliers., which results in a function independent of wholesale price, w, representing a transfer within the SC.

The centralized problem

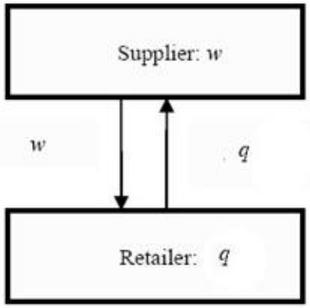
$$\max_{q} J(q) = \max_{q} \{E[ym - h^+ x^+ - h^- x^-] - cq\}$$

s.t.

$$x = q - d,\ q \geq 0.$$

Game theory and Supply chain Management:

- The stocking game.



We first study the centralized problem. Similar to retailer objective function, by determining the expectation of , we obtain

$$\max_q J(q) = \max_q \{$$

$$\int_0^q mDf(D)dD + \int_q^\infty mqf(D)dD - \int_0^q h^+(q-D)f(D)dD - \int_q^\infty h^-(D-q)f(D)dD - cq \}.$$

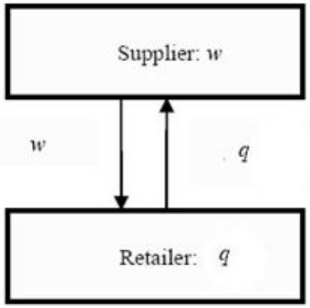
By employing the first-order optimality condition to this function, we have

$$\frac{\partial J(q)}{\partial q} = mqf(q) - mqf(q) + \int_q^\infty mf(D)dD - \int_0^q h^+ f(D)dD + \int_q^\infty h^- f(D)dD - c = 0,$$

which, after simple manipulations, results in

$$m(1 - F(q)) - h^+ F(q) + h^-(1 - F(q)) - c = 0 .$$

Game theory and Supply chain Management:



- The stocking game.

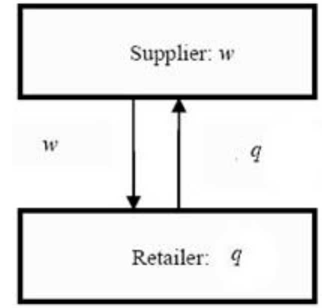
$$F(q^*) = \frac{m + h^- - c}{m + h^- + h^+}.$$

We can also verify the sufficient condition, i.e., that the system wide objective function is concave, i.e.

$$\frac{\partial^2 J(q)}{\partial q^2} = -(m + h^+ + h^-) f(q) \le 0.$$

Let $f(D) > 0$ for $d^{\min} \le D \le d^{\max}$. Then, since ordering less than the minimum demand, $d^{\min}$, as well as more than the maximum demand, $d^{\max}$, does not make any sense, the centralized objective function is strictly concave and thus we find a unique solution.

Game theory and Supply chain Management:



- The stocking game.

Game analysis

The supplier chooses the wholesale price *w and the retailer* selects the order quantity, *q*. The supplier then produces *q* units at unit cost *c* and delivers them to the retailer.

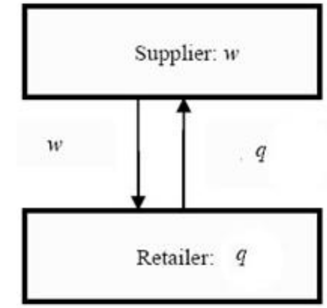
Using the first-order optimality conditions for the retailer's problem, we have

$$\frac{\partial J(q,w)}{\partial q} = mqf(q) - mqf(q) + \int_q^\infty mf(D)dD - \int_0^q h^+ f(D)dD + \int_q^\infty h^- f(D)dD - w = 0$$

Thus, we find that the maximum wholesale price, $w^M = m + h^-$, so that if $w \le w^M$, the best retailer's response is determined by

$$F(q) = \frac{m + h^- - w}{m + h^- + h^+}.$$

Game theory and Supply chain Management:



- The stocking game.

Game analysis

System wide optimal

$$F(q^*) = \frac{m + h^- - c}{m + h^- + h^+}.$$

Game theory (not cooperative)

$$F(q) = \frac{m + h^- - w}{m + h^- + h^+}$$

Proposition 2.6. In vertical co

That is to say non-cooperative decision making of retailer and supplier decreases order quantity, deteriorates service level, and profits of them.

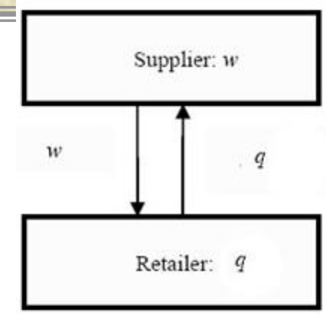
Therefore some kind of mechanisms (coordination mechanisms)should be applied to reduce this undesirable effect which called double margination.

case, the retailer makes no

Game theory and Supply chain Management:

- The stocking game.

Stackelberg equilibrium



Assume that the supplier is a leader in the Stackelberg game. The supplier's objective function with q subject to the optimal retailer's response $q=q^R(w)$ is determined by

$$J_s(q,w) = (w-c)\, q^R(w).$$

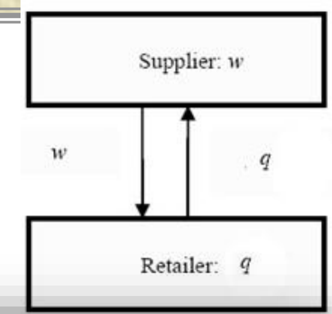
Differentiating the supplier's objective function, we have

$$\frac{\partial J_s(q,w)}{\partial w} = q^R(w) + (w-c)\frac{\partial q^R(w)}{\partial w} = 0 .$$

The value of $\frac{\partial q^R(w)}{\partial w}$ can be calculated from differentiating optimal order quantity in previous nash equilibrium, which result in value.

$$f(q^R(w))\frac{\partial q^R(w)}{\partial w} = -\frac{1}{m + h^- + h^+} .$$

Game theory and Supply chain Management:



- The stocking game.

Stackelberg equilibrium

Proposition 2.7. *Let $f(D)>0$ for $D\geq 0$, otherwise $f(D)=0$. The pair (w^s,q^s), where w^s and $q^s=q^R(w^s)$ satisfy*

$$q^R(w^s)-\frac{w^s-c}{(m+h^-+h^+)f(q^R(w^s))}=0\,,\; F(q^R(w^s))=\frac{m+h^- - w^s}{m+h^-+h^+}\,,$$

constitutes a Stackelberg equilibrium of the stocking game with $c<w^s<m+h^-=w^M$.

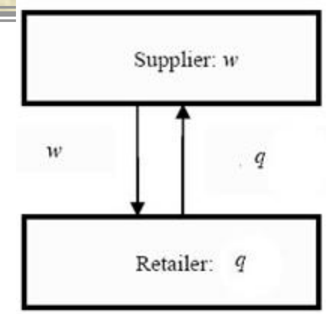
$$\frac{\partial J_s(q,w)}{\partial w}=q^R(w)-\frac{w-c}{(m+h^-+h^+)f(q^R(w))}=0\,,$$

where

$$F(q^R(w))=\frac{m+h^- - w}{m+h^-+h^+}\,.$$

Game theory and Supply chain Management:

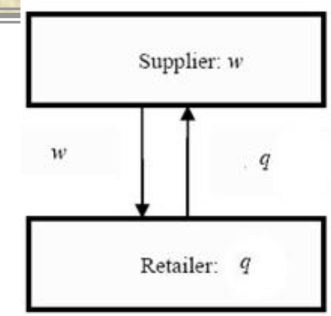
- The stocking game.



Coordination

- As we see earlier, not cooperative strategies (vertical competition) deteriorate SC performance and profits of supplier and retailer.
- Due to the same double marginalization effect, the coordination in this game is similar to that discussed for the pricing game: discounting and profit sharing.
- We show general discounting mechanism

Game theory and Supply chain Management:



- The stocking game.

Coordination

- general discounting mechanism

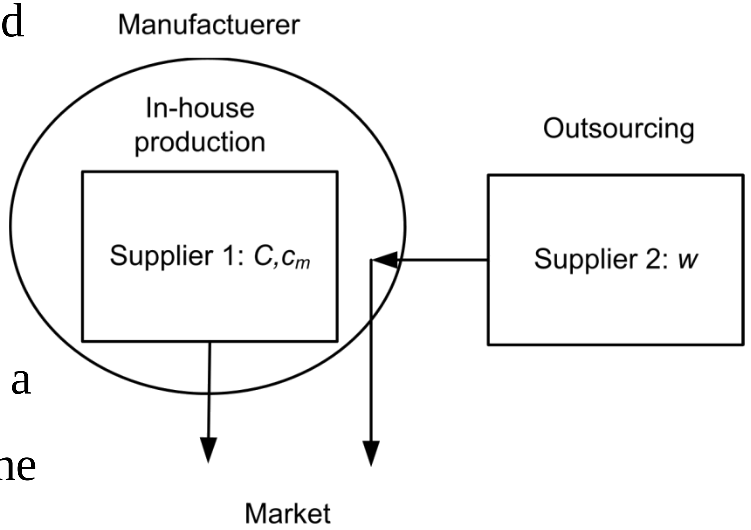
Proposition 2.8. *Let $w(q^*) < w^M$, and the discounting scheme be such that if $w(q)$ is a continuous function of q, $\frac{\partial w(q)}{\partial q} < c$ and $\frac{\partial^2 w(q)}{\partial q^2} \leq 0$ for $q < q^*$, and $\frac{\partial w(q)}{\partial q} \geq c$ for $q > q^*$, then the supplier orders the system-wide optimal quantity q^*.*

$$\frac{\partial w(q)}{\partial q} < c,\ \frac{\partial^2 w(q)}{\partial q^2} \geq 0, \text{ if } q < q^* \text{ and } \frac{\partial w(q)}{\partial q} \geq c, \text{ if } q > q^*.$$

Game theory and Supply chain Management:

- The outsourcing game.

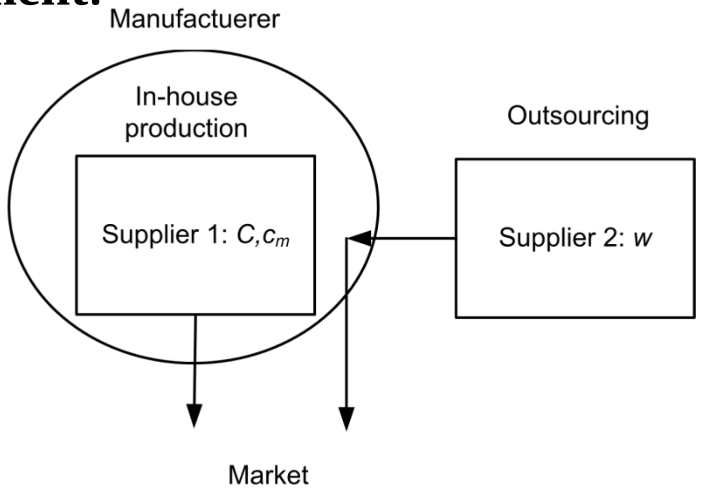
- In this section, the classical, single-period newsvendor model with a setup cost is turned into an outsourcing game.
- We consider a single manufacturer with two potential situations. He either incurs a fixed cost per each production order or the product produced is characterized by frequently changing characteristics and/or technology.



- These changes may be due to new product features and/or technological developments so that each change induces a non-negligible fixed cost.

Game theory and Supply chain Management:

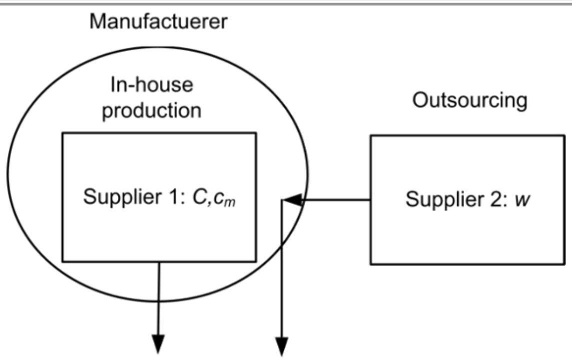
- The outsourcing game.



- the manufacturer has two options.
- One is to order the production in-house, which incurs an irreversible fixed cost C as well as variable cost c_m per unit product.
- The other option involves outsourcing the production to a single supplier. Then the manufacturer incurs only the variable purchasing cost w per product unit and the supplier incurs a unit production cost c.
- We assume that $c>c_m$, no initial inventory, and a profitable in-house production.

Game theory and Supply chain Management:

- The outsourcing game.



The manufacturer's problem

$$\max_q J_m(q,w) =$$

$$\max\{\max_q \{E[ym - h^+x^+ - h^-x^-] - wq\}, \max_q \{E[ym - h^+x^+ - h^-x^-] - c_m q - C\}\},$$

s.t.

$$x = q - d,$$

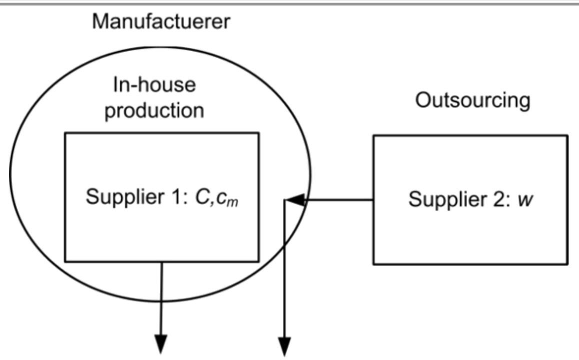
$$q \geq 0,$$

where $x^+=\max\{0, x\}$ and $x^- = \max\{0, -x\}$ are respectively inventory surplus and shortage at the end of a period, and $y=\min\{q,d\}$ is the number of products sold.

The manufacturer's objective function have two parts, the first one represents profit from outsourcing and the second in-house production profit.

Game theory and Supply chain Management:

- The outsourcing game.



- From optimal value of simple newsvendor, it is obvious that the optimal manufacturer's outsourcing order q' (for the first part of objective function) is

$$F(q') = \frac{m + h^- - w}{m + h^- + h^+}.$$

- Similarly, it is obvious that the optimal manufacturer's in-house order q'' (for the second part of objective function) is

$$F(q'') = \frac{m + h^- - c_m}{m + h^- + h^+}.$$

Game theory and Supply chain Management:

- The outsourcing game.

Introduce a cost function, $\pi(q)$, such that

$$\pi(q)=E[ym - h^+x^+ - h^-x^-].$$

Then,

$$\pi(q')\text{-}wq'=$$

$$\int_0^{q'} mDf(D)dD+\int_{q'}^{\infty} mq'f(D)dD-\int_0^{q'} h^+(q'-D)f(D)dD-\int_{q'}^{\infty} h^-(D-q')f(D)dD\,wq'$$

- is the maximum profit if outsourcing is selected. Moreover, the maximum profit when in-house production is selected is:

$$\pi(q'')\text{-}c_m q''\text{-}C=$$

$$\int_0^{q''} mDf(D)dD+\int_{q''}^{\infty} mq''f(D)dD-\int_0^{q''} h^+(q''-D)f(D)dD-\int_{q''}^{\infty} h^-(D-q'')f(D)dD\text{-}c_m q''\text{-}C.$$

Game theory and Supply chain Management:

- The outsourcing game.

Thus, the optimal manufacturer's choice for a given wholesale price is summarized by

$$q = \begin{cases} q', \text{if } \pi(q') - wq' \geq \pi(q'') - c_m q'' - C \\ q'', \text{otherwise}, \end{cases}$$

where q' is the outsourcing order, while q'' is the in-house production

$$F(q') = \frac{m + h^- - w}{m + h^- + h^+}. \qquad F(q'') = \frac{m + h^- - c_m}{m + h^- + h^+}.$$

Game theory and Supply chain Management:

- The outsourcing game.

w^o is the smallest root of the expression below

$$\int_0^{q''} mDf(D)dD+\int_{q''}^{\infty} mq''f(D)dD-\int_0^{q''} h^+(q''-D)f(D)dD-\int_{q''}^{\infty} h^-(D-q'')f(D)dD-c_m q''-C=$$

$$\int_0^{q'} mDf(D)dD+\int_{q'}^{\infty} mq'f(D)dD-\int_0^{q'} h^+(q'-D)f(D)dD-\int_{q'}^{\infty} h^-(D-q')f(D)dD-$$

$$w^o q',$$

where $F(q'')=\dfrac{m+h^- -c_m}{m+h^- +h^+}$ and $F(q')=\dfrac{m+h^- -w^o}{m+h^- +h^+}$.

$$\boxed{q=\begin{cases} q', \text{if } c\le w\le w^0, \\ q'', \text{if } w^0 < c, \end{cases}}$$

where $F(q'')=\dfrac{m+h^- -c_m}{m+h^- +h^+}$ and $F(q')=\dfrac{m+h^- -w^o}{m+h^- +h^+}$.

Game theory and Supply chain Management:

- The outsourcing game.

The supplier's problem

$$\max_{w} J_s(q,w) = (w-c)q$$

s.t.

$$c \le w \le w^0.$$

Note that if $\pi(q'')-c_m q''-C \le \pi(q')-cq'$, then the supplier's problem has a feasible solution. Otherwise, $c_m<w^o<c$, and the supplier's problem has no feasible solution since, in order to compete with in-house production, the supplier has to set the wholesale price below his marginal cost, $w<c$.